AD-A063 880 ROME AIR DEVELOPMENT CENTER GRIFFISS AFB N Y F/G 20/3

MAGNETOSTATIC WAVE TRANSDUCERS WITH VARIABLE COUPLING.(U)

SEP 78 I J WEINBERG, J C SETHARES

UNCLASSIFIED RADC-TR-78-205 NL

| OF |

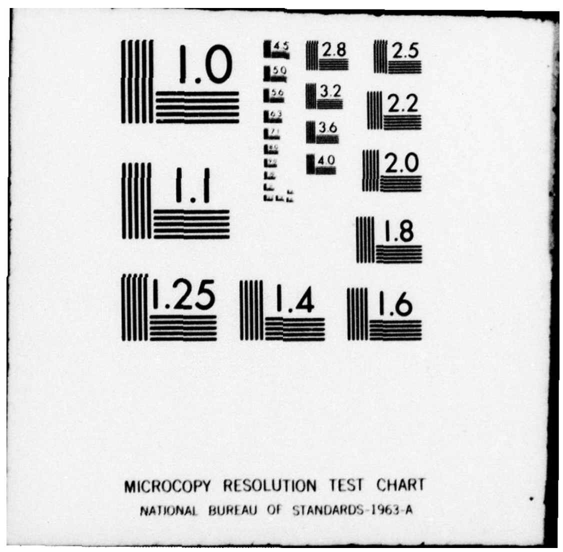
AD A063880

END

DATE FILMED

3-79

DDC


1.0
4.5
5.0
5.6
6.3
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A063880

DDC FILE COPY

(14)

RADC-TR-78-2ϕ5
IN-HOUSE REPORT
SEPTEMBER 1978

(12)
R.S.

LEVEL III

(6)

Magnetostatic Wave Transducers With Variable Coupling,

(10) I. J. WEINBERG
JAMES C. SETHARES

(11) Sep 78

(12) 42p.

(16) 23ϕ5

(17) J5

DDC
RECEIVED
JAN 30 1979
C

Approved for public release; distribution unlimited.

ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

309 050

79 01 26 003

This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-78-205 has been reviewed and is approved for publication.

APPROVED: *Walter Rotman*
WALTER ROTMAN, Chief
Antennas & RF Components Branch
Electromagnetic Sciences Division

APPROVED: *Allan C. Schell*
ALLAN C. SCHELL, Acting Chief
Electromagnetic Sciences Division

FOR THE COMMANDER: *John P. Huss*
JOHN P. HUSS
Acting Chief, Plans Office

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (EEA) Hanscom AFB MA 01731. This will assist us in maintaining a current mailing list.

Do not return this copy. Retain or destroy.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-78-205	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) MAGNETOSTATIC WAVE TRANSDUCERS WITH VARIABLE COUPLING		5. TYPE OF REPORT & PERIOD COVERED In-House Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) I.J. Weinberg James C. Sethares		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Deputy for Electronic Technology (RADC/EEA) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2305J527
11. CONTROLLING OFFICE NAME AND ADDRESS Deputy for Electronic Technology (RADC/EEA) Hanscom AFB Massachusetts 01731		12. REPORT DATE September 1978 13. NUMBER OF PAGES 41
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (*of this Report*) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*) Magnetostatics Transducers Yttrium iron garnet Magnetostatic surface waves		
20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*) Transducer theory for magnetostatic waves in saturated ferrites is extended to include variable coupling. A gap is introduced between transducer and ferrite to adjust the coupling between electromagnetic driving structure and magnetostatic waves. Expressions are developed for the radiation resistance of normal modes of a truncated infinite array and for independent transducer conducting strips of a finite array, as a function of liftoff, for grating and meander transducers. This technology has application in microwave signal processing systems.		

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

79 01 26 021

Contents

Illustrations

Illustrations

Magnetostatic Wave Transducers With Variable Coupling

1. INTRODUCTION

In a previous report,[1] the characteristics of periodic magnetostatic surface wave transducers on the surface of yttrium iron garnet were analyzed. Here, we extend that analysis to include periodic transducers lifted off the surface. The liftoff provides variable coupling between the electromagnetic driving structure and magnetostatic waves. This adjustable coupling is needed for effective signal processing.[2]

The analysis presented differs from that used in Reference 1. Here, we employ the magnetostatic approximation at the outset and introduce a magnetic potential. This procedure allows the same analysis to be used for investigating magnetostatic forward volume waves (MSFVWs). For this reason, we provide a detailed analysis for the present magnetostatic surface waves, so that the study may be adapted in follow-on work to these volume waves. The physical model consists of a thin periodic transducer separated from a YIG slab by a gap, with the entire

---

(Received for publication 19 September 1978)

1. Sethares, J.C., Tsai, T., and Koltunov, I.L. (1978) Periodic Magnetostatic Surface Wave Transducers, RADC-TR-78-78.
2. Emtage, P.R. (1978) Interaction of magnetostatic waves with a current, J. Appl. Phys. 49:4475.

structure sandwiched between two ground planes. Our results reduce to those of Reference 1 when the gap is set equal to zero. When we use the appropriate characteristic equation and permeabilities, our results will be applicable to MSFVWs.[3]

2. BASIC THEORY

2.1 Basic Equations

We first analyze the finite structure with ground planes as shown in Figure 1. A transducer in the form of a meander or grating is excited with an RF current. Figure 2 shows how the transducer is connected to the ground plane structure and to the input/output line. The current establishes RF magnetic fields that generate a variety of propagating modes within the structure.

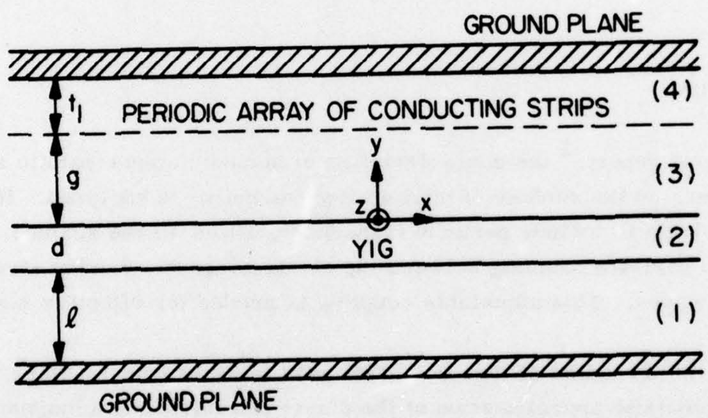


Figure 1. Geometry of the System Composed of YIG Film of Thickness d, Conducting Strips Spaced a Distance g above YIG Surface, and Two Ground Planes

3. Miller, N.D.J. (1977) Non-reciprocal propagation of magnetostatic volume waves, Phys. Stat. 43:593-600.

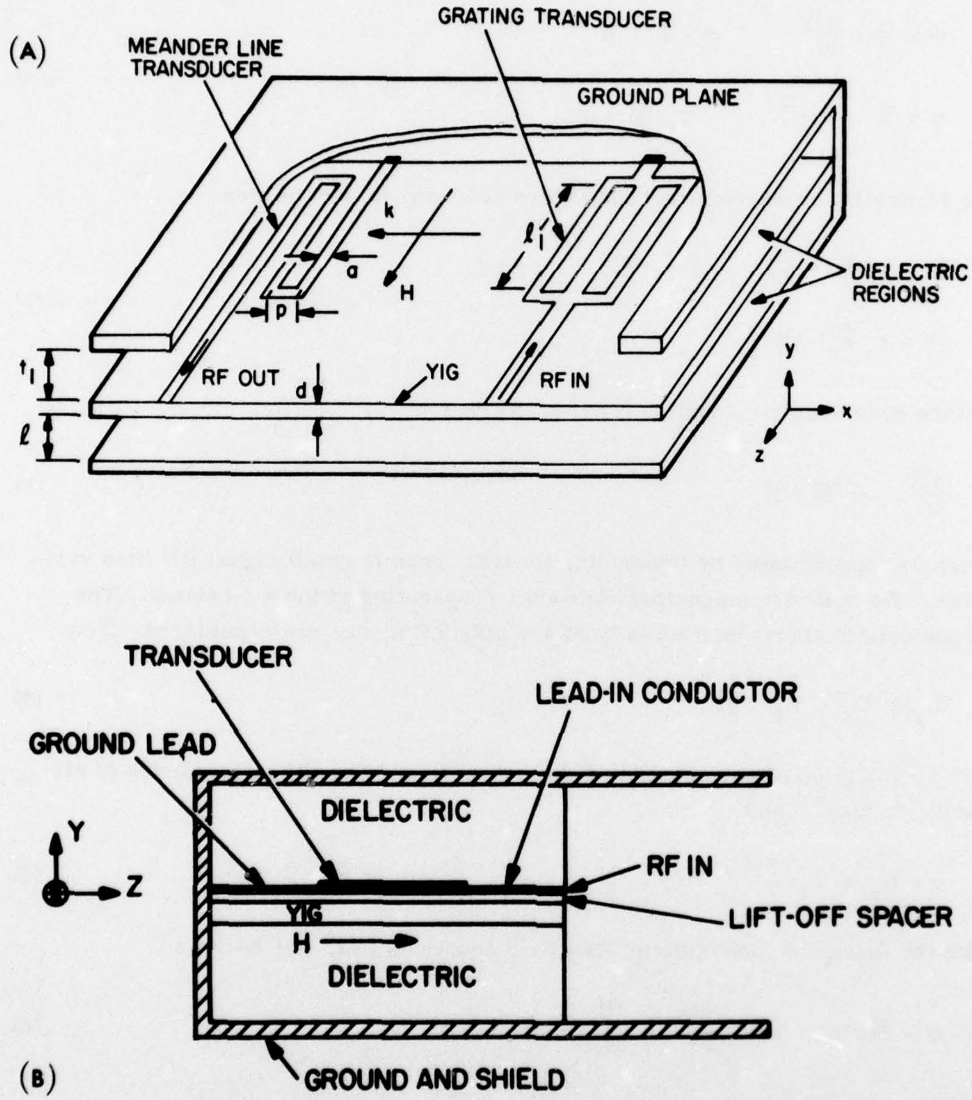


Figure 2. Delay Line Configuration for Magnetostatic Waves. a. A magnetostatic *surface wave delay line configuration showing a grating and meander line transducer structure.* b. Transducer connections to ground plane structures

The problem is analyzed by satisfying Maxwell's equations and the gyromagnetic equation simultaneously, along with appropriate electromagnetic boundary conditions. The equations which are satisfied in each of the four regions (Figure 1)

$$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t} \quad , \quad \nabla \cdot \overline{B} = 0$$
$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \quad , \quad \nabla \cdot \overline{D} = 0 \tag{1}$$

are Maxwell's equations; the constitutive relations in each region

$$\overline{B} = \mu_o(\overline{H} + \overline{M}) = \mu_o \overleftrightarrow{\mu_r} \cdot \overline{H}$$
$$\overline{D} = \epsilon_o \overleftrightarrow{\epsilon_r} \cdot \overline{E} \tag{2}$$

and the gyromagnetic equation for the YIG region

$$\frac{\partial \overline{M}}{\partial t} = -\gamma \, \overline{M} \times \overline{H} \tag{3}$$

which is approximated by linearizing to first order in small signal RF field variables. We consider magnetostatic waves propagating in the x direction. The magnetostatic approximation is used and only TE modes are considered. Thus,

$$H_z = E_x = E_y = 0 \tag{4}$$

with no variation of any quantity in the z direction. The time dependence of all quantities is $e^{i\omega t}$ and

$$\omega \, \epsilon \, E_z \approx 0 \tag{5}$$

With the foregoing assumptions, the field equations [Eq. (1)] become

$$\nabla \times \overline{H} = 0 \quad \text{or} \quad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \tag{6}$$

$$\frac{\partial E_z}{\partial y} = -i \, \omega \, B_x$$
$$\frac{\partial E_z}{\partial x} = i \, \omega \, B_y \tag{7}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \tag{8}$$

with the equation $\nabla \cdot \overline{D} = 0$ automatically satisfied. In all regions except for the YIG region, we take the relations

$$\begin{aligned} B_x &= \mu_o H_x \\ B_y &= \mu_o H_y \end{aligned} \tag{9a}$$

while the linearization assumption reduces Eq. (3) for the YIG region to the form

$$\begin{aligned} B_x &= \mu_o(\mu_{11} H_x - i \mu_{12} H_y) \\ B_y &= \mu_o(i \mu_{12} H_x + \mu_{22} H_y) \end{aligned} \tag{9b}$$

Expressions for the permeability components are given by Emtage[2] for both surface and volume waves.

2.2 Magnetic Potential

Since Eq. (6) is satisfied in each of the four regions, we can find a potential function ψ in each region such that

$$\overline{H} = \nabla\psi \tag{10a}$$

All the quantities of interest are now assumed to be functionally constituted in the form

$$f(x, y, t) = F(y)\, e^{-iKx}\, e^{i\omega t} \tag{10b}$$

Suppressing the time dependence, we assume the ψ dependence in each region to be of the form

$$\psi_j = (A_j\, e^{a_j y} + B_j\, e^{-a_j y})\, e^{-iKx} \qquad a_j > 0;\ j = 1, 2, 3, 4 \tag{11}$$

where the a_j, $j = 1, 2, 3, 4$ are to be determined so that the basic equations, Eqs. (6) to (9), are satisfied while the A_j, B_j, $j = 1, 2, 3, 4$ will be determined to satisfy the boundary conditions which will be presented in the following section.

From Eq. (10a) we find, for each of the four regions

$$H_{x_j} = -i K e^{-iKx} (A_j e^{a_j y} + B_j e^{-a_j y})$$

$$j = 1, 2, 3, 4 \qquad (12)$$

$$H_{y_j} = a_j e^{-iKx} (A_j e^{a_j y} - B_j e^{-a_j y})$$

Thus Eq. (6) is automatically satisfied.

Now, in regions 1, 3, and 4 we have from Eq. (9a)

$$B_{x_j} = -i \mu_o K e^{-iKx} (A_j e^{a_j y} + B_j e^{-a_j y})$$

$$j = 1, 3, 4 \qquad (13)$$

$$B_{y_j} = \mu_o a_j e^{-iKx} (A_j e^{a_j y} - B_j e^{-a_j y})$$

while in region 2 we have from Eq. (9b)

$$B_{x_2} = -\mu_o \mu_{11} i K e^{-iKx} (A_2 e^{a_2 y} + B_2 e^{-a_2 y}) - i \mu_o \mu_{12} a_2 e^{-iKx}$$

$$(A_2 e^{a_2 y} - B_2 e^{-a_2 y}) \qquad (14)$$

$$B_{y_2} = \mu_o \mu_{12} K e^{-iKx} (A_2 e^{a_2 y} + B_2 e^{-a_2 y}) + \mu_o \mu_{22} a_2 e^{-iKx}$$

$$(A_2 e^{a_2 y} - B_2 e^{-a_2 y})$$

We now attempt to satisfy Eq. (8) for each of the four regions. In regions 1, 3, 4 we have, from Eq. (13)

$$\frac{\partial B_{x_j}}{\partial x} = -\mu_o K^2 e^{-iKx} (A_j e^{a_j y} + B_j e^{-a_j y})$$

$$j = 1, 3, 4 \qquad (15)$$

$$\frac{\partial B_{y_j}}{\partial y} = \mu_o a_j^2 e^{-iKx} (A_j e^{a_j y} + B_j e^{-a_j y})$$

so that Eq. (8) is satisfied if

$$a_j = |K| \qquad j = 1, 3, 4 \qquad (16)$$

In region 2 we have, from Eq. (14)

$$\frac{\partial B_{x_2}}{\partial x} = -\mu_o \mu_{11} K^2 e^{-iKx} (A_2 e^{a_2 y} + B_2 e^{-a_2 y}) - \mu_o \mu_{12} a_2 K e^{-iKx} (A_2 e^{a_2 y} - B_2 e^{-a_2 y})$$

$$(17)$$

$$\frac{\partial B_{y_2}}{\partial y} = \mu_o \mu_{12} a_2 K e^{-iKx} (A_2 e^{a_2 y} - B_2 e^{-a_2 y}) + \mu_o \mu_{22} a_2^2 e^{-iKx} (A_2 e^{a_2 y} + B_2 e^{-a_2 y})$$

so that Eq. (8) is satisfied is

$$\mu_{22} a_2^2 = \mu_{11} K^2 \qquad (18)$$

Defining

$$\beta = \sqrt{\frac{\mu_{11}}{\mu_{22}}} \qquad (19)$$

we require

$$a_2 = \beta |K| \qquad (20)$$

for Eq. (8) to be satisfied in region 2.

By integrating both Eqs. (7) in each region and utilizing Eqs. (13) and (14), we have in regions 1, 3, 4

$$E_{z_j} = -i\,\omega \left[-i\,\mu_o \frac{K}{a_j} e^{-iKx} \left(A_j e^{a_j y} - B_j e^{-a_j y}\right)\right]$$

$$j = 1, 3, 4 \tag{21}$$

$$E_{z_j} = i\,\omega \left[i\,\mu_o \frac{a_j}{K} e^{-iKx} \left(A_j e^{a_j y} - B_j e^{-a_j y}\right)\right]$$

which are equal if Eq. (16) is satisfied; and in region 2

$$E_{z_2} = -i\,\omega\, e^{-iKx} \left[-i \frac{K}{a_2} \mu_o \mu_{11} \left(A_2 e^{a_2 y} - B_2 e^{-a_2 y}\right) - i\,\mu_o \mu_{12} \left(A_2 e^{a_2 y} + B_2 e^{-a_2 y}\right)\right]$$

$$\tag{22}$$

$$E_{z_2} = i\,\omega\, e^{-iKx} \left[i\,\mu_o \mu_{12} \left(A_2 e^{a_2 y} + B_2 e^{-a_2 y}\right) + i\,\mu_o \mu_{22} \frac{a_2}{K} \left(A_2 e^{a_2 y} - B_2 e^{-a_2 y}\right)\right]$$

which are equal if Eqs. (19) and (20) are satisfied.

We have thus determined the constants a_j, $j = 1, 2, 3, 4$ for each of the four regions, in order that, Eqs. (6) to (9), the basic equations, are satisfied.

2.3 Boundary Conditions

The physical quantities which are specified due to continuity and boundary conditions (Figure 1) are:

$B_y = 0$	at $y = -(\ell + d)$	(23)
H_x, B_y are continuous	at $y = -d$	(24)
H_x, B_y are continuous	at $y = 0$	(25)
B_y is continuous	at $y = g$	(26)
$B_y = 0$	at $y = t_1 + g$	(27)

The physical quantities H_x, H_y, B_x, B_y are actually to be found in each region by employing the ψ_j of Eq. (11) and integrating in K. For the purpose of determining the constants A_j, B_j, we shall write these quantities *temporarily without the* K integrations.

We now have from Eqs. (12) and (13) using Eqs. (16) and (20)

$$H'_{x_j} = -i K e^{-iKx} (A_j e^{|K|y} + B_j e^{-|K|y}) \qquad j = 1, 3, 4 \tag{28}$$

$$H'_{x_2} = -i K e^{-iKx} (A_2 e^{\beta|K|y} + B_2 e^{-\beta|K|y})$$

and

$$B'_{y_j} = \mu_o|K| e^{-iKx} (A_j e^{|K|y} - B_j e^{-|K|y}) \qquad j = 1, 3, 4 \tag{29}$$

$$B'_{y_2} = \mu_o \mu_{12} K e^{-iKx} (A_2 e^{\beta|K|y} + B_2 e^{-\beta|K|y}) + \mu_o \mu_{22} \beta|K| e^{-iKx} (A_2 e^{\beta|K|y} - B_2 e^{-\beta|K|y})$$

The prime indicates that the quantity has been written without the K integration. By writing

$$\alpha_1 = \sqrt{\mu_{11}\mu_{22}} + \mu_{12} \frac{K}{|K|} = \mu_{22}\beta + s\mu_{12} \tag{30}$$

$$\alpha_2 = \sqrt{\mu_{11}\mu_{22}} - \mu_{12} \frac{K}{|K|} = \mu_{22}\beta - s\mu_{12}$$

we simplify Eq. (29) as

$$B'_{y_j} = \mu_o|K| e^{-iKx} (A_j e^{|K|y} - B_j e^{-|K|y}) \qquad j = 1, 3, 4 \tag{31}$$

$$B'_{y_2} = \mu_o|K| e^{-iKx} (\alpha_1 A_2 e^{\beta|K|y} - \alpha_2 B_2 e^{-\beta|K|y})$$

The eight constants A_j, B_j, $j = 1, 2, 3, 4$ will be found in terms of one remaining constant by using Eqs. (28) and (31) in satisfying the seven boundary conditions in Eqs. (23) to (27). The last constant will then be found by satisfying the additional boundary condition

$$H_{x_4} - H_{x_3} = J(x) \qquad \text{at} \qquad y = g \tag{32}$$

where J(x) is a prescribed current distribution function.

Proceeding with boundary conditions [Eqs. (23) to (27)] systematically from region 1 to region 4 and employing Eqs. (28) and (31), we have

$$\begin{aligned} & A_1\, e^{-|K|(\ell+d)} - B_1\, e^{|K|(\ell+d)} = 0 \\ & A_1\, e^{-|K|d} + B_1\, e^{|K|d} = A_2\, e^{-\beta|K|d} + B_2\, e^{\beta|K|d} \end{aligned} \tag{33}$$

$$\alpha_1 A_2\, e^{-\beta|K|d} - \alpha_2 B_2\, e^{\beta|K|d} = A_1\, e^{-|K|d} - B_1\, e^{|K|d} \tag{34}$$

$$\begin{aligned} & A_3 + B_3 = A_2 + B_2 \\ & A_3 - B_3 = \alpha_1 A_2 - \alpha_2 B_2 \end{aligned} \tag{35}$$

$$\begin{aligned} & A_4\, e^{|K|g} - B_4\, e^{-|K|g} = A_3\, e^{|K|g} - B_3\, e^{-|K|g} \\ & A_4\, e^{|K|(t_1+g)} - B_4\, e^{-|K|(t_1+g)} = 0 \end{aligned} \tag{36}$$

Solving Eq. (33), we obtain

$$\begin{aligned} A_1 &= \frac{e^{|K|(\ell+d)}\,(A_2\, e^{-\beta|K|d} + B_2\, e^{\beta|K|d})}{2\cosh|K|\ell} \\ B_1 &= \frac{e^{-|K|(\ell+d)}\,(A_2\, e^{-\beta|K|d} + B_2\, e^{\beta|K|d})}{2\cosh|K|\ell} \end{aligned} \tag{37}$$

Solving Eq. (36) we obtain

$$A_4 = \frac{-e^{-|K|(t_1+g)}\left(A_3 e^{|K|g} - B_3 e^{-|K|g}\right)}{2 \sinh |K| t_1}$$

$$B_4 = \frac{-e^{|K|(t_1+g)}\left(A_3 e^{|K|g} - B_3 e^{-|K|g}\right)}{2 \sinh |K| t_1} \tag{38}$$

Solving Eq. (35), we obtain

$$A_3 = \frac{1}{2}\left[A_2(1+\alpha_1) + B_2(1-\alpha_2)\right]$$

$$B_3 = \frac{1}{2}\left[A_2(1-\alpha_1) + B_2(1+\alpha_2)\right] \tag{39}$$

By writing

$$T = \frac{\alpha_2 + \tanh |K|\ell}{\alpha_1 - \tanh |K|\ell} \tag{40}$$

we write Eq. (34) as, employing Eq. (37),

$$A_2 = B_2 T e^{2\beta|K|d} \tag{41}$$

The remaining constants are then obtained in terms of B_2. From Eq. (37) we obtain

$$A_1 = \frac{B_2(1+T)}{(1 + e^{-2|K|\ell})} e^{(\beta+1)|K|d}$$

$$B_1 = \frac{B_2(1+T)}{(1 + e^{2|K|\ell})} e^{(\beta-1)|K|d} \tag{42}$$

By writing

$$U = (1 - \alpha_2)\, e^{-\beta|K|d} + (1 + \alpha_1)\, T\, e^{\beta|K|d}$$

$$V = (1 + \alpha_2)\, e^{-\beta|K|d} + (1 - \alpha_1)\, T\, e^{\beta|K|d} \tag{43}$$

we have from Eq. (39)

$$A_3 = \frac{1}{2} B_2\, U\, e^{\beta|K|d}$$

$$B_3 = \frac{1}{2} B_2\, V\, e^{\beta|K|d} \tag{44}$$

and from Eq. (36)

$$A_4 = \frac{B_2\, e^{\beta|K|d}}{2(e^{2|K|t_1} - 1)} \left(V\, e^{-2|K|g} - U\right)$$

$$B_4 = \frac{B_2\, e^{\beta|K|d}}{2(1 - e^{-2|K|t_1})} \left(V - U\, e^{2|K|g}\right) \tag{45}$$

We now have A_1, B_1, A_2, A_3, B_3, A_4, and B_4 defined in terms of the remaining constant B_2 which is to be determined by satisfying the remaining boundary condition [Eq. (32)] where the H_{x_j}, j = 3, 4 is taken as, using Eq. (28).

$$H_{x_j} = -i \int_{-\infty}^{\infty} K e^{-iKx} \left[A_j(K)\, e^{|K|y} + B_j(K)\, e^{-|K|y}\right] dK \qquad j = 3, 4 \tag{46}$$

The A_3, B_3, A_4, B_4 appearing in Eq. (46) are now written as functions of K. The boundary condition [Eq. (32)] thus implies

$$- i \int_{-\infty}^{\infty} K e^{-iKx} \left[A_4(K) e^{|K|g} + B_4(K) e^{-|K|g} \right.$$

$$\left. - A_3(K) e^{|K|g} - B_3(K) e^{-|K|g} \right] dK = J(x) \tag{47}$$

The integration in Eq. (47) can be accomplished by multiplying both sides of the expression by $e^{iK'x}$ and integrating with respect to x from $-\infty$ to ∞ and noting that

$$\int_{-\infty}^{\infty} e^{i(K'-K)x} \, dx = 2\pi\delta(K' - K) \tag{48}$$

where $\delta(K)$ is the Dirac delta function. The expression then becomes

$$\int_{-\infty}^{\infty} K \left[A_4(K) e^{|K|g} + B_4(K) e^{-|K|g} - A_3(K) e^{|K|g} - B_3(K) e^{-|K|g} \right]$$

$$\delta(K' - K) \, dK = \frac{i}{2\pi} \int_{-\infty}^{\infty} J(x) \, e^{iK'x} \, dx \tag{49}$$

Defining, as the Fourier transform

$$\mathfrak{J}(K') = \int_{-\infty}^{\infty} J(x) \, e^{iK'x} \, dx \tag{50}$$

and then replacing K' by K, one notes that expression (49) becomes

$$K \left[A_4(K) e^{|K|g} + B_4(K) e^{-|K|g} - A_3(K) e^{|K|g} - B_3(K) e^{-|K|g} \right] = \frac{i\,\mathfrak{J}(K)}{2\pi} \tag{51}$$

Utilizing Eqs. (43) to (45), we note that the preceding expression becomes

$$\frac{B_2}{2} K e^{\beta|K|d}\left[\coth|K|t_1 \,(V e^{-|K|g} - U e^{|K|g}) - (U e^{|K|g} + V E^{-|K|g})\right]$$

$$= \frac{i\,\tilde{\mathcal{J}}(K)}{2\pi} \tag{52}$$

or

$$\frac{B_2}{2} K e^{\beta|K|d}\left[V e^{-|K|g}(\coth|K|t_1 - 1) - U e^{|K|g}(\coth|K|t_1 + 1)\right]$$

$$= \frac{i\,\tilde{\mathcal{J}}(K)}{2\pi} \tag{53}$$

Defining

$$F(K) = \frac{e^{\beta|K|d}}{2}\left[V e^{-|K|g}(\coth|K|t_1 - 1) - U e^{|K|g}(\coth|K|t_1 + 1)\right] \tag{54}$$

we have

$$B_2 = \frac{i\,\tilde{\mathcal{J}}(K)}{2\pi K F(K)} \tag{55}$$

The other constants are now found [Eq. (41) to (45)] as

$$A_2 = \frac{i\,\tilde{\mathcal{J}}(K)\, T\, e^{2\beta|K|d}}{2\pi K F(K)} \tag{56}$$

$$A_1 = \frac{i\,\tilde{\mathcal{J}}(K)(1 + T)\, e^{(\beta+1)|K|d}}{2\pi K F(K)(1 + e^{-2|K|\ell})}$$

$$B_1 = \frac{i\,\tilde{\mathcal{J}}(K)(1 + T)\, e^{(\beta-1)|K|d}}{2\pi K F(K)(1 + e^{2|K|\ell})} \tag{57}$$

$$A_3 = \frac{i\,\tilde{J}(K)\,U\,e^{\beta|K|d}}{4\pi\,K\,F(K)}$$

$$B_3 = \frac{i\,\tilde{J}(K)\,V\,e^{\beta|K|d}}{4\pi\,K\,F(K)} \tag{58}$$

$$A_4 = \frac{i\,\tilde{J}(K)(V\,e^{-2|K|g} - U)\,e^{\beta|K|d}}{4\pi\,K\,F(K)(e^{2|K|t_1} - 1)}$$

$$B_4 = \frac{i\,\tilde{J}(K)(V - U\,e^{2|K|g})\,e^{\beta|K|d}}{4\pi\,K\,F(K)(1 - e^{-2|K|t_1})} \tag{59}$$

2.4 Field Equations

With the determination of all the constants, we have the time dependence suppressed expressions for H_{x_j} and B_{y_j}, $j = 1, 2, 3, 4$ in terms of integration in K, from Eqs. (28) and (31) using Eqs. (55) to (59), as

$$H_{x_1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{\beta|K|d}(T+1)\,\tilde{J}(K)}{F(K)}\,\frac{\cosh\,[|K|(\ell + d + y)]}{\cosh\,|K|\ell}\,e^{-iKx}\,dK$$

$$B_{y_1} = \frac{i\,\mu_o}{2\pi}\int_{-\infty}^{\infty} \frac{s\,e^{\beta|K|d}(T+1)\,\tilde{J}(K)}{F(K)}\,\frac{\sinh\,[|K|(\ell + d + y)]}{\cosh\,|K|\ell}\,e^{-iKx}\,dK \tag{60}$$

$$H_{x_2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{\beta|K|d}\,\tilde{J}(K)}{F(K)}\left[T\,e^{\beta|K|(d+y)} + e^{-\beta|K|(d+y)}\right]e^{-iKx}\,dK$$

$$B_{y_2} = \frac{i\,\mu_o}{2\pi}\int_{-\infty}^{\infty} \frac{s\,e^{\beta|K|d}\,\tilde{J}(K)}{F(K)}\left[\alpha_1\,T\,e^{\beta|K|(d+y)} - \alpha_2\,e^{-\beta|K|(d+y)}\right]e^{-iKx}\,dK \tag{61}$$

$$H_{x_3} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\beta|K|d}\,\tilde{J}(K)}{2F(K)} \left[U\, e^{|K|y} + V\, e^{-|K|y}\right] e^{-iKx}\, dK$$

$$B_{y_3} = \frac{i\,\mu_o}{2\pi} \int_{-\infty}^{\infty} \frac{s\, e^{\beta|K|d}\,\tilde{J}(K)}{2F(K)} \left[U\, e^{|K|y} - V\, e^{-|K|y}\right] e^{-iKx}\, dK \tag{62}$$

$$H_{x_4} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\beta|K|d}\,\tilde{J}(K) \cosh[|K|(g + t_1 - y)]}{2F(K) \sinh |K|t_1} \left(U\, e^{|K|g} - V\, e^{-|K|g}\right) e^{-iKx}\, dK$$

$$B_{y_4} = \frac{i\,\mu_o}{2\pi} \int_{-\infty}^{\infty} \frac{s\, e^{\beta|K|d}\,\tilde{J}(K) \sinh[|K|(g + t_1 - y)]}{2F(K) \sinh |K|t_1} \left(U\, e^{|K|g} - V\, e^{-|K|g}\right) e^{-iKx}\, dK \tag{63}$$

where $s = K/|K|$ and writing

$$F_T(K) = e^{-2\beta|K|d}\, F(K) \tag{64}$$

we have from Eqs. (54) and (43)

$$F_T(K) = \frac{1}{2} \Big\{ (\coth |K|t_1 - 1) \left[(1 + \alpha_2)\, e^{-2\beta|K|d} + (1 - \alpha_1)T\right] e^{-|K|g} - (\coth |K|t_1 + 1) \left[(1 - \alpha_2)\, e^{-2\beta|K|d} + (1 + \alpha_1)T\right] e^{|K|g} \Big\} \tag{65}$$

The integrals in Eqs. (60) to (63) are evaluated by contour integration. The integrals are assumed to vanish on the infinite upper and lower semicircles due to the behavior of $\tilde{J}(K)$. There are residues at the two real simple zeros of $F_T(K)$ which we denote by

$$F_T(K_s) = 0 \qquad s = -1, 1 \tag{66}$$

The residue is then the remaining portion of the integrand evaluated at K_s multiplied by the reciprocal of $[(\partial/\partial K) F_T(K)]_{K=K_s}$ which we write as

$$F_T^{(1)}(K_s) \equiv \left[\frac{\partial}{\partial K} F_T(K) \right]_{K=K_s} \tag{67}$$

The value of each integral is then $2\pi i$ multiplied by the residues at K_s, $s = -1, 1$. Defining

$$G_s = \frac{\tilde{J}(K_s)\, e^{-\beta |K_s| d}}{F_T^{(1)}(K_s)} \tag{68}$$

$$T_s = \frac{\alpha_2 + \tanh |K_s| \ell}{\alpha_1 - \tanh |K_s| \ell} \tag{69}$$

we can rewrite Eqs. (60) to (64), using one pole at a time, as

$$H_{x_1}^{(s)} = \frac{i\, G_s\, (T_s + 1) \cosh [|K_s| (\ell + d + y)]}{\cosh |K_s| \ell}\, e^{-iK_s x}\,,$$

$$s = -1, 1 \tag{70}$$

$$B_{y_1}^{(s)} = \frac{-\mu_o\, s\, G_s (T_s + 1) \sinh [|K_s| (\ell + d + y)]}{\cosh |K_s| \ell}\, e^{-iK_s x}$$

$$H_{x_2}^{(s)} = i\, G_s\, (T_s\, e^{\beta |K_s| (d+y)} + e^{-\beta |K_s| (d+y)})\, e^{-iK_s x}$$

$$s = -1, 1 \tag{71}$$

$$B_{y_2}^{(s)} = -\mu_o\, s\, G_s\, (\alpha_1 T_s\, e^{\beta |K_s| (d+y)} - \alpha_2\, e^{-\beta |K_s| (d+y)})\, e^{-iK_s x}$$

$$H_{x_3}^{(s)} = \frac{i\,G_s}{2}\left(U_s\, e^{|K_s|y} + V_s\, e^{-|K_s|y}\right) e^{-iK_s x}$$

$$s = -1, 1 \qquad (72)$$

$$B_{y_3}^{(s)} = \frac{-s\,\mu_o\, G_s}{2}\left(U_s\, e^{|K_s|y} - V_s\, e^{-|K_s|y}\right) e^{-iK_s x}$$

$$H_{x_4}^{(s)} = \frac{-i\,G_s \cosh\left[|K_s|(g + t_1 - y)\right]}{2\sinh|K_s|t_1}\left(U_s\, e^{|K_s|g} - V_s\, e^{-|K_s|g}\right) e^{-iK_s x}$$

$$s = -1, 1 \quad (73)$$

$$B_{y_4}^{(s)} = \frac{-\mu_o\, s\, G_s \sinh\left[|K_s|(g + t_1 - y)\right]}{2\sinh|K_s|t_1}\left(U_s\, e^{|K_s|g} - V_s\, e^{-|K_s|g}\right) e^{-iK_s x}$$

where, from Eq. (43)

$$U_s = (1 - \alpha_2)\, e^{-\beta|K_s|d} + (1 + \alpha_1) T_s\, e^{\beta|K_s|d}$$

$$(74)$$

$$V_s = (1 + \alpha_2)\, e^{-\beta|K_s|d} + (1 - \alpha_1) T_s\, e^{\beta|K_s|d}$$

There remains to find $\partial/\partial K[F_T(K)]$ which is $F_T^{(1)}(K)$ where $F_T(K)$ is given by Eq. (65). When differentiating, we can consider α_1 and α_2 to be independent of K. Then from Eq. (40) we find

$$\frac{\partial T}{\partial K} = \frac{s\,\ell\,(\alpha_1 + \alpha_2)\,\mathrm{sech}^2\,|K|\,\ell}{(\alpha_1 - \tanh|K|\ell)^2} \qquad (75)$$

We now obtain, by differentiating Eq. (65) and utilizing Eq. (43)

$$\begin{aligned} 2\,\frac{\partial}{\partial K} F_T(K) = {} & e^{-\beta|K|d}\left(U\, e^{|K|g} - V\, e^{-|K|g}\right) s\, t_1\, \mathrm{csch}^2\,|K|t_1 \\ & - \left[(\coth|K|\,t_1 + 1)\, U\, e^{|K|g} + (\coth|K|t_1 - 1)\, V\, e^{-|K|g}\right] s\, g\, e^{-\beta|K|d} \\ & + \left[(\coth|K|\,t_1 + 1)\, e^{|K|g}(1 - \alpha_2) - (\coth|K|t_1 - 1)\, e^{-|K|g}(1 + \alpha_2)\right] \\ & \quad 2\,\beta\, s\, d\, e^{-2\beta|K|d} \end{aligned}$$

$$+ [(\coth |K| t_1 - 1)\, e^{-|K| g} (1 - \alpha_1) - (\coth |K| t_1 + 1)$$

$$e^{|K| g} (1 + \alpha_1)] \; \frac{s\,\ell\, (\alpha_1 + \alpha_2) \operatorname{sech}^2 |K| \ell}{(\alpha_1 - \tanh |K| \ell)^2} \tag{76}$$

enabling the computation of $F_T^{(1)}(K_s)$ in Eq. (68).

2.5 Magnetostatic Wave Power

The magnetostatic wave power for each K_s value and for a width ℓ_1 is given by

$$P^{(s)} = \frac{\ell_1}{2} \int_{-(\ell+d)}^{t_1+g} E_z^{(s)}\, \overline{H_y^{(s)}}\, dy \tag{77}$$

where $\overline{H_y^{(s)}}$ denotes the complex conjugate of $H_y^{(s)}$.

From Eq. (7) we obtain, for all regions,

$$E_{z_j} = -\frac{\omega}{K} B_{y_j} = -s \frac{\omega}{|K|} B_{y_j} \qquad j = 1, 2, 3, 4 \tag{78}$$

From Eq. (9) we have

$$B_{y_j} = \mu_o H_{y_j} \qquad j = 1, 3, 4 \tag{79}$$

$$B_{y_2} = \mu_o (i\, \mu_{12} H_{x_2} + \mu_{22} H_{y_2})$$

$$H_{y_2} = \frac{1}{\mu_{22}} (B_{y_2}/\mu_o - i\, \mu_{12} H_{x_2}) \tag{80}$$

Equation (77) is then separated by regions as

$$P^{(s)} = \frac{\ell_1}{2}\left[\int_{-(\ell+d)}^{-d} E_{z_1}^{(s)}\overline{H_{y_1}^{(s)}}\,dy + \int_{-d}^{o} E_{z_2}^{(s)}\overline{H_{y_2}^{(s)}}\,dy + \int_{o}^{g} E_{z_3}^{(s)}\overline{H_{y_3}^{(3)}}\,dy\right.$$

$$\left. + \int_{g}^{t_1+g} E_{z_4}^{(s)}\overline{H_{y_4}^{(s)}}\,dy\right] \tag{81}$$

We evaluate each of the integrals in Eq. (81) using Eqs. (78) to (80).

In region 1, we have from Eqs. (78) and (79)

$$E_{z_1}^{(s)}\overline{H_{y_1}^{(s)}} = \frac{-s\omega\mu_o}{|K_s|}\left|H_{y_1}^{(s)}\right|^2 \tag{82}$$

Thus, utilizing Eqs. (70) and (79), we obtain

$$\int_{-(\ell+d)}^{-d} E_{z_1}^{(s)}\overline{H_{y_1}^{(s)}}\,dy = \frac{-s\omega\mu_o G_s^2 (T_s+1)^2}{|K_s| \cosh^2 |K_s|\ell}\int_{-(\ell+d)}^{-d} \sinh^2 [|K_s|(\ell+d+y)]\,dy \tag{83}$$

Since

$$\sinh^2 U = \frac{1}{2}(\cosh 2U - 1) \tag{84}$$

we have

$$\int_{-(\ell+d)}^{-d} E_{z_1}^{(s)}\overline{H_{y_1}^{(s)}}\,dy = \frac{-s\omega\mu_o G_s^2 (T_s+1)^2}{|K_s| \cosh^2 |K_s|\ell}\left(\frac{\sinh 2|K_s|\ell}{4|K_s|} - \frac{\ell}{2}\right) \tag{85}$$

In region 4, we also have from Eqs. (78) and (79)

$$E_{z_4}^{(s)}\overline{H_{y_4}^{(s)}} = \frac{-s\omega\mu_o}{|K_s|}\left|H_{y_4}^{(s)}\right|^2 \tag{86}$$

and from Eqs. (74) and (79)

$$\int_g^{t_1+g} E_{z_4}^{(s)} \overline{H_{y_4}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2 \left(U_s e^{|K_s|g} - V_s e^{-|K_s|g}\right)^2}{4|K_s| \sinh^2 |K_s| t_1}$$

$$\int_g^{t_1+g} \sinh^2 [|K_s|(g + t_1 - y)]\, dy \tag{87}$$

and utilizing Eq. (84) we have

$$\int_g^{t_1+g} E_{z_4}^{(s)} \overline{H_{y_4}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2 \left(U_s e^{|K_s|g} - V_s e^{-|K_s|g}\right)^2}{4|K_s| \sinh^2 |K_s| t_1}$$

$$\left(\frac{\sinh 2|K_s| t_1}{4|K_s|} - \frac{t_1}{2}\right) \tag{88}$$

In region 3, we again have from Eqs. (78) and (79)

$$E_{z_3}^{(s)} \overline{H_{y_3}^{(s)}} = \frac{-s\omega\mu_o}{|K_s|} |H_{y_3}^{(s)}|^2 \tag{89}$$

and from Eqs. (72) and (79)

$$\int_o^g E_{z_3}^{(s)} \overline{H_{y_3}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2}{4|K_s|} \int_o^g \left(U_s^2 e^{2|K_s|y} + V_s^2 e^{-2|K_s|y} - 2 U_s V_s\right) dy \tag{90}$$

Thus

$$\int_o^g E_{z_3}^{(s)} \overline{H_{y_3}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2}{4|K_s|^2} \left[\frac{U_s^2}{2} (e^{2|K_s|g} - 1) - \frac{V_s^2}{2} (e^{-2|K_s|g} - 1) - 2 U_s V_s |K_s| g\right] \tag{91}$$

In region 2, we have from Eqs. (78) and (80)

$$E^{(s)}_{z_2} \overline{H^{(s)}_{y_2}} = \frac{-s\omega\mu_o}{|K_s|} \left(i\,\mu_{12} H^{(s)}_{x_2} \overline{H^{(s)}_{y_2}} + \mu_{22} \left|H^{(s)}_{y_2}\right|^2 \right) \tag{92}$$

Now, from Eqs. (71) and (80)

$$H^{(s)}_{y_2} = \frac{1}{\mu_{22}} \Big[-s\,G_s \left(\alpha_1 T_s e^{\beta|K_s|(d+y)} - \alpha_2 e^{-\beta|K_s|(d+y)} \right)$$

$$+ \mu_{12} G_s \left(T_s e^{\beta|K_s|(d+y)} + e^{-\beta|K_s|(d+y)} \right) \Big] e^{-iK_s x} \tag{93}$$

which simplifies to

$$H^{(s)}_{y_2} = \frac{G_s}{\mu_{22}} \left[T_s e^{\beta|K_s|(d+y)} (\mu_{12} - \alpha_1 s) + e^{-\beta|K_s|(d+y)} (\mu_{12} + \alpha_2 s) \right] e^{-iK_s x} \tag{94}$$

Now from Eq. (30)

$$\begin{aligned} \mu_{12} - \alpha_1 s &= \mu_{12} - s\,(\mu_{22}\,\beta + s\,\mu_{12}) = -s\,\mu_{22}\,\beta \\ \mu_{12} + \alpha_2 s &= \mu_{12} + s\,(\mu_{22}\,\beta - s\,\mu_{12}) = s\,\mu_{22}\,\beta \end{aligned} \tag{95}$$

and

$$H^{(s)}_{y_2} = s\,\beta\,G_s \left(-T_s e^{\beta|K_s|(d+y)} + e^{-\beta|K_s|(d+y)} \right) e^{-iK_s x} \tag{96}$$

Since from Eq. (92)

$$\int_{-d}^{o} E^{(s)}_{z_2} \overline{H^{(s)}_{y_2}}\, dy = \frac{-s\omega\mu_o}{|K_s|} \left[i\,\mu_{12} \int_{-d}^{o} H^{(s)}_{x_2} \overline{H^{(s)}_{y_2}}\, dy + \mu_{22} \int_{-d}^{o} \left|H^{(s)}_{y_2}\right|^2 dy \right] \tag{97}$$

we have first, from Eq. (96)

$$\mu_{22} \int_{-d}^{0} |H_{y_2}^{(s)}|^2 \, dy = \mu_{22} \beta^2 G_s^2 \int_{-d}^{0} \left(- T_s e^{\beta|K_s|(d+y)} + e^{-\beta|K_s|(d+y)} \right)^2 dy \tag{98}$$

or

$$\mu_{22} \int_{-d}^{0} |H_{y_2}^{(s)}|^2 \, dy = \mu_{22} \beta^2 G_s^2 \left[\frac{T_s^2}{2\beta|K_s|} (e^{2\beta|K_s|d} - 1) - \frac{(e^{2\beta|K_s|d} - 1)}{2\beta|K_s|} - 2 \, T_s \, d \right] \tag{99}$$

We next have, from Eqs. (96) and (71)

$$i \mu_{12} \int_{-d}^{0} H_{x_2}^{(s)} \overline{H_{y_2}^{(s)}} \, dy = - \mu_{12} G_s^2 \, s \, \beta \int_{-d}^{0} \left(T_s e^{\beta|K_s|(d+y)} + e^{-\beta|K_s|(d+y)} \right) \left(- T_s e^{\beta|K_s|(d+y)} + e^{-\beta|K_s|(d+y)} \right) dy \tag{100}$$

or

$$i \mu_{12} \int_{-d}^{0} H_{x_2}^{(s)} \overline{H_{y_2}^{(s)}} \, dy = -\mu_{12} G_s^2 \, s \, \beta \int_{-d}^{0} \left(e^{-2\beta|K_s|(d+y)} - T_s^2 e^{2\beta|K_s|(d+y)} \right) dy \tag{101}$$

which becomes

$$i \mu_{12} \int_{-d}^{0} H_{x_2}^{(s)} \overline{H_{y_2}^{(s)}} \, dy = \frac{-\mu_{12} G_s^2 \, s}{2|K_s|} \left[\left(1 - e^{-2\beta|K_s|d} \right) - T_s^2 \left(e^{2\beta|K_s|d} - 1 \right) \right] \tag{102}$$

Thus the integral in Eq. (97) becomes, using Eqs. (99) and (102)

$$\int_{-d}^{o} E_{z_2}^{(s)} \overline{H_{y_2}^{(s)}} \, dy = \frac{-s\omega\mu_o G_s^2}{2|K_s|^2} \Big[T_s^2 (e^{2\beta|K_s|d} - 1)(\beta\mu_{22} + \mu_{12} s)$$

$$+ (e^{-2\beta|K_s|d} - 1)(-\beta\mu_{22} + \mu_{12} s) - 4\beta^2 \mu_{22} |K_s| T_s d \Big] \tag{103}$$

Utilizing Eq. (30), we have

$$\int_{-d}^{o} E_{z_2}^{(s)} \overline{H_{y_2}^{(s)}} \, dy = \frac{-s\omega\mu_o G_s^2}{2|K_s|^2} \Big[\alpha_1 T_s^2 (e^{2\beta|K_s|d} - 1) - \alpha_2 (e^{-2\beta|K_s|d} - 1)$$

$$- 4\beta^2 \mu_{22} |K_s| T_s d \Big] \tag{104}$$

Finally, placing Eqs. (85), (88), (91) and (104) into Eq. (81), we obtain

$$P^{(s)} = \frac{-s\omega\mu_o}{2|K_s|^2} \frac{\ell_1}{2} G_s^2 \Bigg\{ \frac{(T_s + 1)^2}{\cosh^2 |K_s|\ell} \left(\frac{\sinh 2|K_s|\ell}{2} - |K_s|\ell \right)$$

$$+ \frac{\left(U_s e^{|K_s|g} - V_s e^{-|K_s|g} \right)^2}{4 \sinh^2 |K_s| t_1} \left(\frac{\sinh 2|K_s|t_1}{2} - |K_s| t_1 \right)$$

$$+ \frac{1}{2} \left[\frac{U_s^2}{2} (e^{2|K_s|g} - 1) - \frac{V_s^2}{2} (e^{-2|K_s|g} - 1) - 2 U_s V_s |K_s| g \right]$$

$$+ \left[\alpha_1 T_s^2 (e^{2\beta|K_s|d} - 1) - \alpha_2 (e^{-2\beta|K_s|d} - 1) - 4\beta^2 \mu_{22} |K_s| T_s d \right] \Bigg\} \tag{105}$$

Defining

$$A^{(s)} = \frac{(T_s + 1)^2}{\cosh^2 |K_s|\ell} \left(\frac{\sinh 2|K_s|\ell}{4} - \frac{|K_s|\ell}{2} \right) + \frac{\left(U_s\, e^{|K_s|g} - V_s\, e^{-|K_s|g} \right)^2}{4 \sinh^2 |K_s| t_1}$$

$$\left(\frac{\sinh 2|K_s|t_1}{4} - \frac{|K_s|t_1}{2} \right)$$

$$+ \frac{1}{4} \left[\frac{U_s^2}{2} (e^{2|K_s|g} - 1) - \frac{V_s^2}{2} (e^{-2|K_s|g} - 1) - 2\, U_s\, V_s\, |K_s| g \right]$$

$$+ \left[\frac{\alpha_1}{2} T_s^2 (e^{2\beta|K_s|d} - 1) - \frac{\alpha_2}{2} (e^{-2\beta|K_s|d} - 1) - 2\, \beta^2 \mu_{22} |K_s|\, T_s\, d \right] \tag{106}$$

we can write

$$P^{(s)} = \frac{-s\,\omega\,\mu_o\, \ell_1\, G_s^2}{2|K_s|^2} A^{(s)} \tag{107}$$

for the magnetostatic surface wave power due to each K_s, $s = -1, 1$. By writing

$$P^{(s)} = \frac{1}{2} [\tilde{J}(K_s)]^2 R_o^{(s)} = \frac{1}{2} [\tilde{J}(K_s)]^2 \ell_1 R_1^{(s)} \tag{108}$$

where

$$R_o^{(s)} = \ell_1 R_1^{(s)}$$

we obtain, using Eqs. (68) and (107)

$$R_1^{(s)} = \frac{-s\,\omega\,\mu_o\, e^{-2\beta|K_s|d}}{|K_s|^2 [F_T^{(1)}(K_s)]^2} A^{(s)} \tag{109}$$

It is to be noted that all the results obtained here reduce to the results obtained in Reference 1 when the gap region, region 3, is removed and g is set to 0. We will consider Eq. (109), the radiation resistance, again in more detail in Section 4.

3. FREE SPACE CASE: NO GROUND PLANES

In this section we determine the results for the case of infinite free space where the ℓ in region 1 and the t_1 in region 4 are permitted to become infinitely large.

We first find that boundary conditions [Eqs. (23) and (27)] need to be modified to

$$B_y = 0 \qquad y \to \infty \tag{110}$$

$$B_y = 0 \qquad y \to -\infty \tag{111}$$

This causes Eqs. (33) and (36) to change to

$$\begin{aligned} B_1 &= 0 \\ A_1 &= A_2\, e^{(1-\beta)|K|d} + B_2\, e^{(1+\beta)|K|d} \end{aligned} \tag{112}$$

And

$$\begin{aligned} A_4 &= 0 \\ B_4 &= B_3 - A_3\, e^{2|K|g} \end{aligned} \tag{113}$$

Now redefining

$$T = \frac{\alpha_2 + 1}{\alpha_1 - 1} \tag{114}$$

We again have, from Eqs. (34), (35), (39) and (43), as in (41) and (44)

$$A_2 = e^{2\beta|K|d}\; T\; B_2 \tag{115}$$

$$A_3 = \frac{U}{2} e^{\beta|K|d} B_2$$

$$B_3 = \frac{V}{2} e^{\beta|K|d} B_2 \tag{116}$$

and from Eqs. (112) and (113)

$$A_1 = B_2 (1 + T) e^{(1+\beta)|K|d} \tag{117}$$

$$B_4 = \frac{B_2}{2} e^{\beta|K|d} (V - U e^{2|K|g}) \tag{118}$$

Now utilizing boundary condition (32) in a similar manner as was done earlier, we obtain

$$-B_2 K U e^{|K|g} e^{\beta|K|d} = \frac{i}{2\pi} \tilde{\mathfrak{J}}(K) \tag{119}$$

The complete set of constants, with

$$F(K) = -U e^{|K|g} e^{\beta|K|d} \tag{120}$$

are given as

$$A_1 = \frac{i \tilde{\mathfrak{J}}(K) e^{(1+\beta)|K|d} (1 + T)}{2\pi K F(K)}$$

$$B_1 = 0 \tag{121}$$

$$A_2 = \frac{i \tilde{\mathfrak{J}}(K) T e^{2\beta|K|d}}{2\pi K F(K)}$$

$$B_2 = \frac{i \tilde{\mathfrak{J}}(K)}{2\pi K F(K)} \tag{122}$$

$$A_3 = \frac{i\,\tilde{J}(K)\,e^{\beta|K|d}\,U}{4\pi\,K\,F(K)}$$

$$B_3 = \frac{i\,\tilde{J}(K)\,e^{\beta|K|d}\,V}{4\pi\,K\,F(K)} \tag{123}$$

$$A_4 = 0$$

$$B_4 = \frac{i\,\tilde{J}(K)\,e^{\beta|K|d}}{4\pi\,K\,F(K)}\,(V - U\,e^{2|K|g}) \tag{124}$$

which lead to the modified field equations in regions 1 and 4

$$H_{x_1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{(T+1)\,e^{\beta|K|d}\,\tilde{J}(K)}{F(K)}\,e^{|K|(d+y)}\,e^{-iKx}\,dK$$

$$B_{y_1} = \frac{i\,\mu_o}{2\pi}\int_{-\infty}^{\infty} \frac{s(T+1)\,e^{\beta|K|d}\,\tilde{J}(K)}{F(K)}\,e^{|K|(d+y)}\,e^{-iKx}\,dK \tag{125}$$

$$H_{x_4} = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\tilde{J}(K)\,e^{\beta|K|d}}{2F(K)}\,(U\,e^{|K|g} - V\,e^{-|K|g})\,e^{|K|(g-y)}\,e^{-iKx}\,dK$$

$$B_{y_4} = \frac{i\,\mu_o}{2\pi}\int_{-\infty}^{\infty} \frac{s\,\tilde{J}(K)\,e^{\beta|K|d}}{2F(K)}\,(U\,e^{|K|g} - V\,e^{-|K|g})\,e^{|K|(g-y)}\,e^{-iKx}\,dK \tag{126}$$

while for the other regions the expressions remain as in Eqs. (61) and (62). Defining as before and using Eq. (120), we have

$$F_T(K) = e^{-2\beta|K|d}\,F(K) = -\,e^{-\beta|K|d}\,e^{|K|g}\,U \tag{127}$$

We perform the contour integrations as before.

We denote the two real simple zeros of $F_T(K)$ by K_s, $s = -1, 1$ so that here

$$U_s = 0 \tag{128}$$

where U_s is as earlier defined in Eq. (74) and, by Eq. (114)

$$T_s = T \tag{129}$$

Again denoting

$$F_T^{(1)}(K_s) \equiv \left[\frac{\partial}{\partial K} F_T(K)\right]_{K=K_s} \tag{130}$$

and defining

$$G_s = \frac{\tilde{J}(K_s)\, e^{-\beta|K_s|d}}{F_T^{(1)}(K_s)} \tag{131}$$

we have, as the result of the contour integrations, for regions 1 and 4

$$\begin{aligned} H_{x_1}^{(s)} &= i\, G_s\, (T_s + 1)\, e^{|K_s|(d+y)}\, e^{-iK_s x} \\ & \qquad\qquad\qquad\qquad s = -1,\ 1 \\ B_{y_1}^{(s)} &= -\, s\, \mu_o\, G_s\, (T_s + 1)\, e^{|K_s|(d+y)}\, e^{-iK_s x} \end{aligned} \tag{132}$$

and

$$\begin{aligned} H_{x_4}^{(s)} &= \frac{-i\, G_s}{2}\, (U_s\, e^{|K_s|g} - V_s\, e^{-|K_s|g})\, e^{|K_s|(g-y)}\, e^{-iK_s x} \\ & \qquad\qquad\qquad\qquad s = -1,\ 1 \\ B_{y_4}^{(s)} &= \frac{-s\, \mu_o\, G_s}{2}\, (U_s\, e^{|K_s|g} - V_s\, e^{-|K_s|g})\, e^{|K_s|(g-y)}\, e^{-iK_s x} \end{aligned} \tag{133}$$

with the expressions for the other regions being the same as Eqs. (71) and (72). Writing out $F_T(K)$ as

$$F_T(K) = -e^{|K|g}\left[(1 - \alpha_2)\, e^{-2\beta|K|d} + (1 + \alpha_1)\, T\right] \tag{134}$$

and noting from (114) that here

$$\frac{\partial T}{\partial K} = 0 \tag{135}$$

we obtain for the derivative

$$\frac{\partial}{\partial K} F_T(K) = - s\, g\, e^{|K|g} \left[(1 - \alpha_2)\, e^{-2\beta|K|d} + (1 + \alpha_1)\, T \right]$$

$$+ 2\beta s d\, e^{|K|g} (1 - \alpha_2)\, e^{-2\beta|K|d} \tag{136}$$

Setting $K = K_s$ in Eq. (136) and noting Eq. (128), we obtain

$$F_T^{(1)}(K_s) = 2 s \beta d\, e^{|K_s|g} (1 - \alpha_2)\, e^{-2\beta|K_s|d} \tag{137}$$

The magnetostatic surface wave power for each K_s value and width ℓ_1 is now given by

$$P^{(s)} = \frac{\ell_1}{2} \int_{-\infty}^{\infty} E_z^{(s)}\, \overline{H_y^{(s)}}\, dy \tag{138}$$

which when broken down by regions, becomes

$$P^{(s)} = \frac{\ell_1}{2} \left[\int_{-\infty}^{-d} E_{z_1}^{(s)}\, \overline{H_{y_1}^{(s)}}\, dy + \int_{-d}^{0} E_{z_2}^{(s)}\, \overline{H_{y_2}^{(s)}}\, dy + \int_{0}^{g} E_{z_3}^{(s)}\, \overline{H_{y_3}^{(s)}}\, dy \right.$$

$$\left. + \int_{g}^{\infty} E_{z_4}^{(s)}\, \overline{H_{y_4}^{(s)}}\, dy \right] \tag{139}$$

We employ the relations [Eqs. (78) to (80)] for E_{z_j} and H_{y_j}, j = 1, 2, 3, 4 which enter into Eq. (139).

In region 1, we have the analogous relation to Eq. (83) by utilizing Eq. (125)

$$\int_{-\infty}^{-d} E_{z_1}^{(s)} \overline{H_{y_1}^{(s)}}\, dy = \frac{-s\omega\mu_o}{|K_s|} G_s^2 (T_s + 1)^2 \int_{-\infty}^{-d} e^{2|K_s|(d+y)}\, dy \tag{140}$$

Thus

$$\int_{-\infty}^{-d} E_{z_1}^{(s)} \overline{H_{y_1}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2 (T_s + 1)^2}{2|K_s|^2} \tag{141}$$

Similarly, in region 4 we have analogous to Eq. (87), utilizing Eq. (126)

$$\int_{g}^{\infty} E_{z_4}^{(s)} \overline{H_{y_4}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2}{4|K_s|} \left(U_s e^{|K_s|g} - V_s e^{-|K_s|g}\right)^2 \int_{g}^{\infty} e^{2|K_s|(g-y)}\, dy \tag{142}$$

which becomes, employing Eq. (128)

$$\int_{g}^{\infty} E_{z_4}^{(s)} \overline{H_{y_4}^{(s)}}\, dy = \frac{-s\omega\mu_o G_s^2 V_s^2 e^{-2|K_s|g}}{8|K_s|^2} \tag{143}$$

In region 3, Eq. (91) obtained earlier holds subject to Eq. (128). Thus

$$\int_{o}^{g} E_{z_3}^{(s)} \overline{H_{y_3}^{(s)}}\, dy = \frac{s\omega\mu_o G_s^2 V_s^2 (e^{-2|K_s|g} - 1)}{8|K_s|^2} \tag{144}$$

while in region 2, Eq. (104) holds exactly as before.

The insertion of Eqs. (141), (143), (144) and (104) into Eq. (139) gives, after cancellation

$$P^{(s)} = \frac{-s\omega\mu_o}{2|K_s|^2} \frac{\ell_1}{2} G_s^2 \Bigg[(T_s + 1)^2 + \frac{V_s^2}{4} + \alpha_1 T_s^2 (e^{2\beta|K_s|d} - 1) - \alpha_2 (e^{-2\beta|K_s|d} - 1) - 4\beta^2 \mu_{22} |K_s| T_s d \Bigg] \tag{145}$$

By making use of Eqs. (74) and (114), one obtains after considerable algebra

$$(T_s + 1)^2 + \frac{V_s^2}{4} + \alpha_1 T_s^2 \left(e^{+2\beta|K_s|d} - 1\right) - \alpha_2 \left(e^{-2\beta|K_s|d} - 1\right) = \frac{U_s^2}{4} \tag{146}$$

which vanishes by virtue of Eq. (128). Thus we have

$$P^{(s)} = \frac{s\omega\mu_o \ell_1}{|K_s|} G_s^2 \beta^2 \mu_{22} T_s d \tag{147}$$

By noting Eqs. (131) and (137), we observe that $e^{-2|K_s|g}$ is a factor in Eq. (147), as expected.

Defining

$$A^{(s)} = -2\beta^2 \mu_{22} |K_s| T_s d \tag{148}$$

we write

$$P^{(s)} = \frac{-s\omega\mu_o \ell_1 G_s^2}{2|K_s|^2} A^{(s)} \tag{149}$$

Writing

$$P^{(s)} = \frac{1}{2}[\tilde{J}(K_s)]^2 R_o^{(s)} \tag{150}$$

we have

$$R_o^{(s)} = \frac{-s\omega\mu_o \ell_1 e^{-2\beta|K_s|d}}{|K_s|^2 [F_T^{(1)}(K_s)]^2} A^{(s)} \tag{151}$$

which agrees with our previous result, Eq. (109).

The free space case is useful because $|K_s|$ can be written as a function of ω and solved directly. It provides insights and serves as a check on the more general case.

4. RADIATION RESISTANCE

In this section, expressions are given for radiation resistance along with computer results for several cases of interest.

4.1 Isolated Independent Conductors

Consider a transducer made up of N thin conducting strips each carrying a spatially uniform current I_o. When the strips are connected in series, the total current I_T flowing into the transducer is $I_T = I_o$. For strips connected in parallel, forming a grating, $I_T = N\, I_o$. Following a previous analysis[1, 4] one obtains

$$R_m^{(s)} = \left[\frac{2R_1^{(s)}\,\ell_1}{(1-\eta)+(1+\eta)N^2}\right]\left[\operatorname{sinc}\left(\frac{a\,K_s}{2\pi}\right)\right]^2 \left|\frac{1-\eta^N e^{iK_s pN}}{1-\eta\, e^{iK_s p}}\right|^2 \qquad (152)$$

where $R_1^{(s)}$ is given by Eq. (109). It is independent of transducer geometry. Equation (152) gives the radiation resistance for a meander or grating array which is made up of N independent conducting strips.

Figure 3 shows plots obtained from Eq. (152) of the radiation resistance per unit width for grating transducers of 1, 2, 3, and 4 independent conducting strips. It gives the radiation resistance for a wave propagating in the $\overline{H} \times \overline{n}$ direction, where n is normal to the surface. The local maxima near 3650 MHz corresponds to the longest wavelength which matches the grating periodicity.

The effects of liftoff are shown in Figure 4 where radiation resistance for a four element grating transducer is plotted for three values of g. The decay is nearly exponential when the ground planes are many wavelengths away. When they are close, the decay is a complicated function of transducer geometry and ground plane spacing.

Figure 5 shows radiation resistance for a meander line. Note the change in vertical scale. There are eight conducting strips connected in series. This produces higher values of resistance than when they are connected in parallel. Radiation resistance for both positive and negative going waves are shown with the nonreciprocity evident. The successive peaks correspond to MSSW wavelengths: $\lambda = n\,p$ with n = 1, 3, 5 and 7.

Figure 5 was obtained from Eq. (152). In the next section a normal mode approach is employed to obtain radiation resistance for the same transducer for the n = 1, 3 normal modes.

---

4. Sethares, J.C. (1978) Magnetostatic Surface Wave Transducers, 78 IEEE MTT MTT-S, Cat. No. 78 CH1355-7, 444.

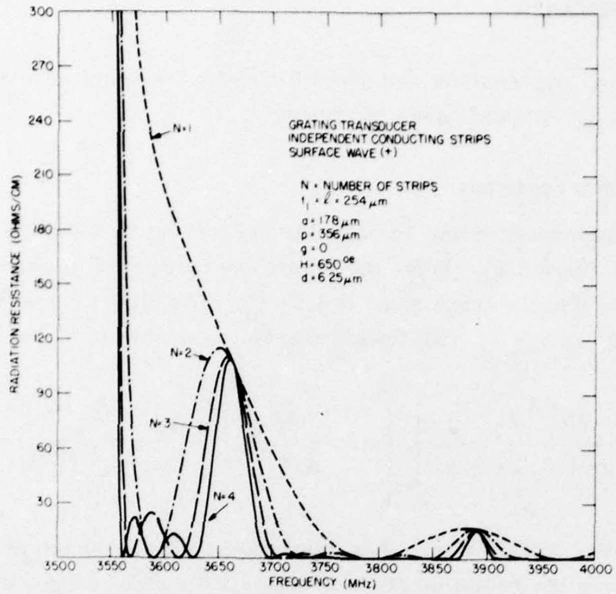


Figure 3. Radiation Resistance for Grating Transducers Having a Different Number of Parallel Independent Conducting Strips

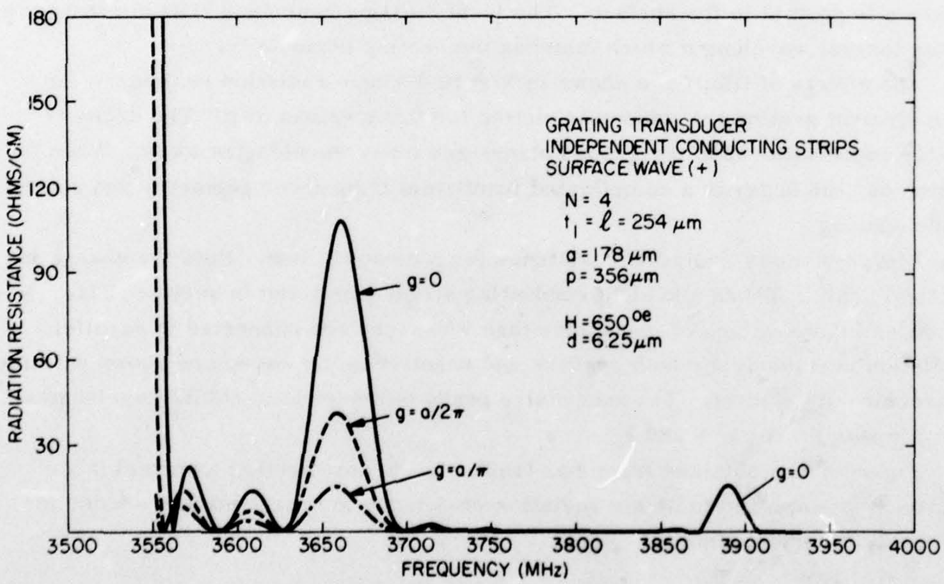


Figure 4. Radiation Resistance as a Function of Liftoff

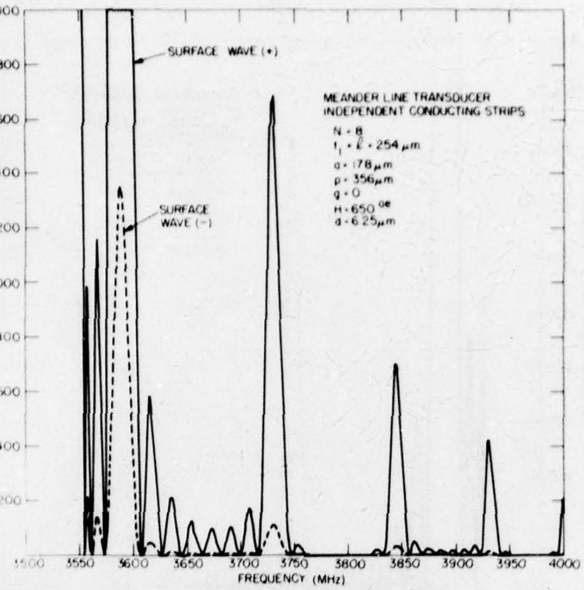


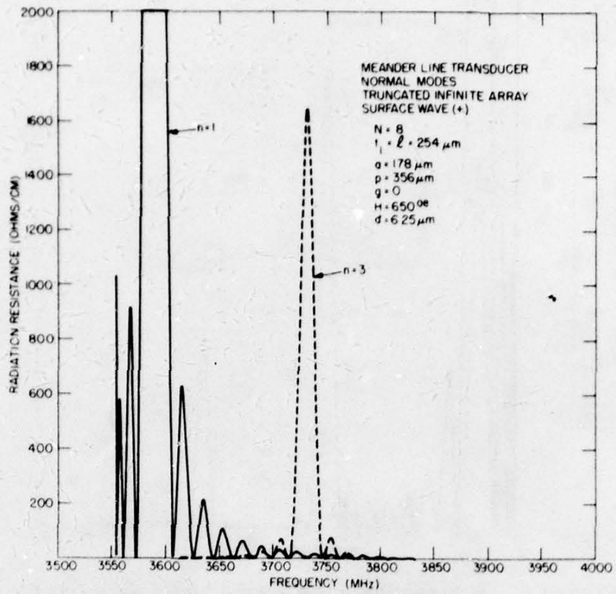
Figure 5. A Meander Line Transducer Showing the Nonreciprocal Radiation Resistance of Positive and Negative Magnetostatic Surface Waves

4.2 Truncated Infinite Array

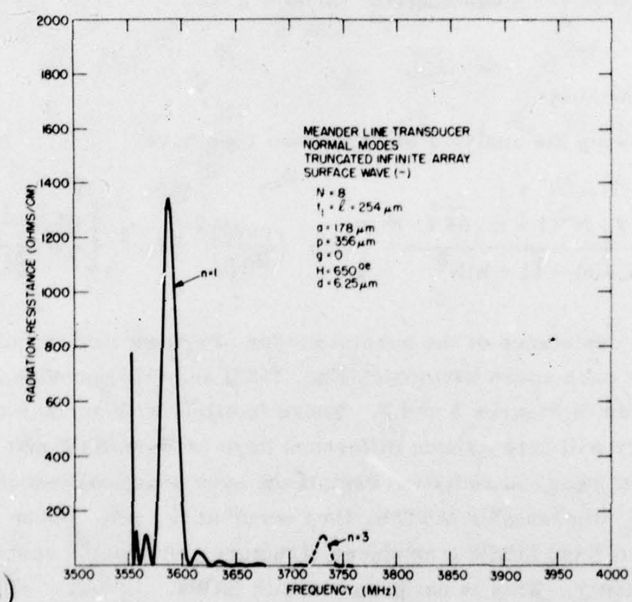
Again, following the analysis of Reference 1 we have

$$R_m^{(s)} = \frac{R_1^{(s)}\,\ell_1\,N^2(1+\eta\cos\ell'\pi)}{(1-\eta)+(1+\eta)N^2}\left[\text{sinc}\,\left(\frac{\ell' a}{2p}\right)\right]^2 \text{sinc}^2\left[\frac{(K_s-\frac{\ell'\pi}{p})Np}{2\pi}\right] \quad (153)$$

for the radiation resistance of the normal modes of a truncated infinite array. For frequencies near each space harmonic, Eqs. (152) and (153) provide nearly identical curves as seen in Figures 5 and 6. Those familiar with surface acoustic wave transducer theory will note a basic difference here between SAW and MSSWs. For SAWs, subsequent peaks in radiation resistance over practical frequency ranges occur at $\omega = n\,\omega_o$ whereas for MSSWs, they occur at $K = n\,K_o$ where n is an integer. This means that a fixed MSSW transducer structure can provide spatial filtering at almost any frequency. This is not possible with SAWs.



(A)



(B)

Figure 6. Normal Mode Radiation Resistance for a Meander Line Transducer. a. Positive waves, b. Negative waves

5. CONCLUSION

Periodic magnetostatic surface wave transducer theory has been extended to include variable coupling between MSSW and EM waves. Variable coupling was achieved by introducing a gap between the YIG surface and transducer. The analysis is given in sufficient detail to allow one to follow the approach used and assumptions made, providing a basis for further extensions of the theory. The technology has application in signal processing directly at microwave frequencies.